

On the origin of the holographic principle

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It was recently suggested that quantum mechanics and gravity are not fundamental but emerge from information loss at causal horizons. On the basis of the formalism the holographic principle is also shown to arise naturally from the loss of information about bulk fields observed by an outside observer. As an application, Witten's prescription is derived.

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I. INTRODUCTION

The holographic principle [1, 2], including the AdS/CFT correspondence [3], asserts an unexpected connection between the physics in a bulk and quantum field theory (QFT) on its boundary surface. The origin of this mysterious connection is still unknown. On the other hand, many studies of black hole physics after Bekenstein and Hawking have implied a deep connection between gravity and thermodynamics [4]. For example, Jacobson suggested that Einstein's equation describes thermodynamics at local Rindler horizons, and Padmanabhan [5] proposed that classical gravity can be derived from the equipartition energy of horizons. Verlinde recently proposed an intriguing idea [6] linking both Einstein's gravity and Newton's mechanics to entropic forces. All these works emphasized the strange connection between thermodynamics and gravity the origin of which is also still mysterious [7–22]. Since the thermodynamic entropy can be interpreted as a measure of information or information loss, the connection implies a close relationship between information and gravity.

In a series of works [23–26], based on information theory, my colleagues and I suggested that information loss (or quantum entanglement) at causal horizons is the key to understanding the origins of dark energy [23], black hole mass [24] and even Einstein's gravity [26]. Many other studies in quantum information science have supported the idea that information is fundamental. For example, one can obtain a quantum mechanical state discrimination bound from the condition that information propagation is not superluminal (see, for example, [27]). Landauer's principle in quantum information science implies that information is physical [28]. Zeilinger and Brukner [29, 30] suggested that quantum randomness arises from the discreteness of information. 't Hooft also suggested that quantum mechanics has a deterministic theory that includes local information loss [31].

Since the number of degrees of freedom (DOF) in a bulk theory is proportional to the volume of the bulk, whereas the number of DOF in a boundary theory is proportional to its surface area, it seems impossible to prove a generic holographic principle within the context of QFT. It might be possible to prove the principle on the basis of new principles that can explain the origin of both gravity and quantum mechanics.

In [32] I showed that quantum mechanics is not fundamental but emerges from information loss at causal horizons. If gravity and quantum mechanics can be derived by considering information loss at causal horizons (see [33] for a review), it is natural to think that the holographic principle has a similar origin. This paper suggests that the principle can be derived by applying information theory to causal horizons.

In Sec. II, the derivation of quantum mechanics from information theory is reviewed. In Sec. III, a derivation of the holographic principle is presented, and Witten's prescription is derived as an application. Sec. IV contains conclusions.

II. QUANTUM FIELD THEORY FROM INFORMATION LOSS

In this section, I review the way in which QFT, and hence quantum mechanics, arises from the application of information theory at causal horizons. It is important to understand that our theory is based on neither QFT nor Einstein's gravity. On the contrary, they could be derived from the postulates below.

Inspired by the works described in Sec. I, we can choose the following postulates as new general guiding principles on which any physical law should be based.

1. *General equivalence: All systems of reference (coordinates) are equivalent for formulating physical laws regardless of their motions.*
2. *Information has a finite density and speed; the quantity of information contained in a finite object is finite, and there is a maximal speed of classical information propagation, namely, the light velocity.*
3. *Information is fundamental: Physical laws regarding an object (matter or spacetime) should be such that they respect observers' information about the object.*

We also assume the metric nature of spacetime (however, we do not assume Einstein's equation) and ignore any fluctuation of spacetime in this paper. Postulate 2 implies that for a given observer, there could be causal horizons that block information about a region the observer cannot access.

Although postulates 1 and 2 are familiar, postulate 3 deserves more explanation. It implies that interpretation of a physical reality depends on the information an observer can access, and hence there is no objective reality independent of observers. This sounds counterintuitive, but it is exactly what ordinary quantum mechanics says. For example, it is possible that a pure qubit state $\psi = (|0\rangle + |1\rangle)/\sqrt{2}$ seen by one observer can be a maximally mixed state for another observer who could not access information about the state. An important point here is that both descriptions of the

same qubit are perfectly valid and not in contradiction. Similarly, regardless of measurements done by an observer inside a causal horizon, quantum states of matter inside the horizon seen by an outside observer are maximally mixed. Two descriptions of the matter including the observer's state itself should be coincident. On average, no observer has priority. Considering the surface action terms in relativity, Padmanabhan also pointed out that physical theories must be formulated in terms of variables any given observer can access [34].

One special conclusion derived from the three postulates together is that physics inside a causal horizon should respect (or be consistent with) the ignorance of an outside observer about the inside region. This naturally introduces the notion of a horizon entropy arising by definition from information loss. Some authors have argued that this information loss is the origin of black hole entropy [35].

Using the above postulates, I showed in [32] that quantum mechanics is not fundamental but emerges from the application of classical information theory to causal horizons. The path integral (PI) quantization and quantum randomness can be derived by considering information loss for accelerating observers of fields or particles crossing Rindler horizons. This implies that information is one of the fundamental roots of all physical phenomena. I also investigated the connection between this theory and Verlinde's entropic gravity theory.

Let us briefly review the information theoretic formalism suggested in Ref. [32]. Consider an accelerating observer Θ_R with acceleration a in the x_1 direction in a flat spacetime with coordinates $x = (t, x_1, x_2, x_3)$ (See Fig. 1). The Rindler coordinates $\xi = (\eta, r, x_2, x_3)$ for the observer are

$$t = r \sinh(a\eta), \quad x_1 = r \cosh(a\eta). \quad (1)$$

There is another observer Θ_M inside the Rindler horizon. Now, consider a field ϕ crossing the Rindler horizon and entering the future wedge F . The configuration for $\phi(x)$ in F is just a scalar function of the coordinates x , not a classical field.

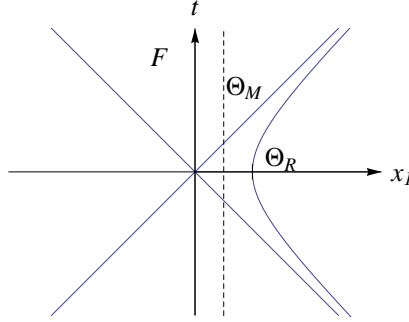


FIG. 1. A Rindler observer Θ_R (curved line) has no accessible information about the field ϕ in a causally disconnected region F . The observer can only estimate a probabilistic distribution of the field, which turns out to be equal to that of a quantum field seen by a Minkowski observer Θ_M (dashed line).

As the field enters the Rindler horizon for the observer Θ_R , the observer receives no further information about future configurations of ϕ . All that the observer can guess about the evolution of ϕ is the probabilistic distribution $P[\phi]$ of ϕ beyond the horizon. The information already known about ϕ constrains $P[\phi]$. According to our postulates, the physics in the wedge F is determined not by deterministic classical physics but by the evolution of information. The maximum ignorance about the field can be mathematically expressed by maximizing the Shannon information entropy $h[P] = -\sum_{i=1}^n P[\phi_i] \ln P[\phi_i]$ of the possible (discrete) configurations $\{\phi_i(x)\}$, $i = 1 \cdots n$ that the field may take with a probability $P[\phi_i]$. If there is information for Θ_R represented by N_j testable expectations

$$\langle f_j \rangle \equiv \sum_{i=1}^n P[\phi_i] f_j[\phi_i], \quad (j = 1 \cdots N_j), \quad (2)$$

we should use Boltzmann's theorem of maximum entropy to calculate the probability distribution $P[\phi]$. Here, f_j , ($k = 1 \cdots N_j$) is a functional of ϕ and $\langle f_j \rangle$ is its statistical expectation value with respect to $P[\phi]$. According to the theorem, by maximizing the Shannon entropy with the constraints in Eq. (2), one can obtain the following probability distribution

$$P[\phi_i] = \frac{1}{Z} \exp \left[- \sum_{j=1}^{N_j} \lambda_j f_j(\phi_i) \right] \quad (3)$$

with a partition function $Z = \sum_{i=1}^n \exp \left[- \sum_{j=1}^{N_j} \lambda_j f_j(\phi_i) \right]$.

One constraint may come from the energy conservation $\sum_{i=1}^n P[\phi_i] H(\phi_i) = E$, where $H(\phi_i)$ is the Hamiltonian as a function of the field configuration ϕ_i and E is its expectation, and another trivial one is the unity of the probabilities $\sum_{i=1}^n P[\phi_i] = 1$. Then, the probability with the constraints estimated by the Rindler observer should be

$$P[\phi_i] = \frac{1}{Z} \exp [-\beta H(\phi_i)], \quad (4)$$

where β is a Lagrangian multiplier, and the partition function is $Z = \sum_{i=1}^n \exp [-\beta H(\phi_i)] = \text{tr } e^{-\beta H}$. Thus, the thermal nature of quantum fields is a natural consequence of classical information theory, when information loss with constraints occurs.

As an example, let us consider a scalar field with Hamiltonian

$$H(\phi) = \int d^3x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] \quad (5)$$

and a potential V . For the Rindler observer with the coordinates (η, r, x_2, x_3) the proper time variance is $ar d\eta$ and the Hamiltonian becomes

$$H_R = \int_{r>0} dr dx_{\perp} ar \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \eta} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{2} (\nabla_{\perp} \phi)^2 + V(\phi) \right], \quad (6)$$

where \perp denotes the plane orthogonal to (η, r) plane. Then, Z becomes Eq. (2.5) of Ref. [36];

$$Z_R = \text{tr } e^{-\beta H_R}. \quad (7)$$

Notice that Z (and hence Z_R) here is not a quantum partition function but a statistical one corresponding to the uncertain field configurations beyond the horizon.

The equivalence of this form of Z_R and a quantum partition function for a scalar field in the Minkowski spacetime (say Z_Q) is shown in Ref. [36], which is the famous Unruh effect. (See [37] for a review.) A continuous version of Eq. (7) in QFT is

$$Z_R = N_0 \int_{\phi(0)=\phi(\beta)} D\phi \exp \left\{ - \int_0^{\beta} d\tilde{\eta} \int_{r>0} dr dx_{\perp} ar H_R \right\}.$$

By further changing integration variables as $\tilde{r} = r \cos(a\tilde{\eta})$, $\tilde{t} = r \sin(a\tilde{\eta})$ and choosing $\beta = 2\pi/a$ the region of integration is transformed into the full two dimensional flat space, which leads to Unruh temperature $T_U = a\hbar/2\pi k_B$, where k_B is the Boltzman constant. Then, the partition function becomes that of an euclidean flat spacetime;

$$\begin{aligned} Z_Q^E &= N_1 \int D\phi \exp \left\{ - \int d\tilde{r} d\tilde{t} dx_{\perp} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tilde{t}} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial \tilde{r}} \right)^2 + \frac{1}{2} (\nabla_{\perp} \phi)^2 + V(\phi) \right] \right\} \\ &= N_1 \int D\phi \exp \left\{ - \frac{I_E}{\hbar} \right\}. \end{aligned} \quad (8)$$

where I_E is the Euclidean action for the scalar field in the inertial frame. Since both of Z_R and Z_Q can be obtained from Z_Q^E by analytic continuation, they are physically equivalent [37]. Thus, the conventional QFT formalism is equivalent to the purely information theoretic formalism for loss of information about field configurations beyond the Rindler horizon.

Recall that Eq. (7) was derived without using any quantum physics. Since quantum mechanics can be thought of as the single particle limit of QFT, this implies that quantum mechanics emerges from the application of information theory to Rindler horizons and is not fundamental in our formalism.

Near any static horizon having more generic static metrics $ds^2 = -f^2 dt^2 + \gamma_{\alpha\beta} dx^{\alpha} dx^{\beta}$, the metric reduces to the Rindler form [38] $ds^2 \simeq -f^2 dt^2 + df^2/\kappa^2 + dL_{\perp}^2$, where κ is the surface gravity and dL_{\perp}^2 is the metric for the orthogonal direction. Therefore, we expect the information theoretic interpretation of QFT to be valid for more generic static metrics.

This information theoretic approach is more than a reinterpretation of quantum mechanics. For example, it could explain the origin of quantum randomness and PI quantization, which were assumptions in ordinary quantum mechanics. Note that by extremizing Z_R this approach also explains the origin of the thermodynamic relation $dE = TdS$ in gravitational systems [33], which leads to entropic gravity and Jacobson's thermodynamic model of Einstein's gravity. Another bonus is the explanation of the well-known analogy between QFT and classical statistical mechanics; QFT is essentially a statistical system in disguise. Surprisingly, this formalism also gives rise to a derivation of the holographic principle, which will be presented in the next section.

III. HOLOGRAPHIC PRINCIPLE FROM INFORMATION LOSS

The information theoretic derivation of quantum mechanics in the previous section makes it simple to understand the physical origin of the holographic principle. Consider a $d + 1$ -dimensional bulk region Ω with a d -dimensional boundary $\partial\Omega$ that is a one-way causal horizon (see Fig. 2) such as a black hole horizon, Rindler horizon or cosmic horizon. Imagine an outside observer Θ_O (like Θ_R) who can not access information about matter or spacetime in the region because of the horizon. The situation in Ω is maximally uncertain to the outside observer, and the best the observer could do is to estimate the probability of each possible field configuration of ϕ in Ω , which turns out to be the probability amplitude in the PI [32]. During this estimation, Θ_O would use the maximal information available to her/him. The previous section showed that the observer's ignorance leads to quantum fluctuations of fields in Ω . Thus, paradoxically, the outside observer's ignorance is an essential ingredient for any physics in Ω .

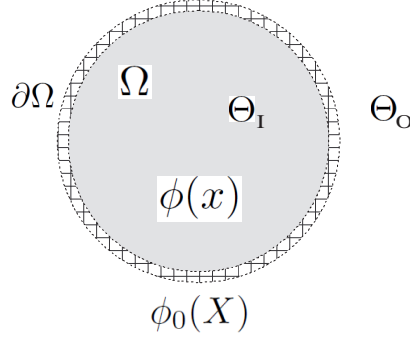


FIG. 2. Consider a bulk Ω with a causal horizon $\partial\Omega$ and an inside observer Θ_I . The outside observer Θ_O has no information about the field configuration $\phi(x)$ in Ω except for its boundary values $\phi_0(X)$ and derivatives. Thus, according to the information theoretic interpretation, the physics in Ω is completely described by the boundary physics on $\partial\Omega$, which is just the holographic principle.

According to the postulate 2, there is no non-local interaction that might allow super-luminal communication. Therefore, we restrict ourselves to local field theory in this paper. For a local field, any influence on Ω from the outside of the horizon should pass the horizon. This means that, according to postulate 3, all the physics in the bulk Ω is fully described by the DOF on the boundary $\partial\Omega$, which is just the essence of the holographic principle! In other words, information loss due to a horizon gives rise to quantum randomness in the bulk, and at the same time allows the outside observer Θ_O to describe the physics in the bulk using only the DOF on the boundary. That is the best Θ_O can do by any means, and the general equivalence principle demands that this description is sufficient for understanding the physics in the bulk, which is the holographic principle.

Therefore, the following version of the holographic principle is a natural consequence of the information theoretic formalism of QFT based on the three postulates.

Theorem 1 (holographic principle). *For local field theory, physics inside a causal horizon can be described completely by physics on the horizon.*

One may think of this as a *derivation*, albeit simple one, of the holographic principle from the information theoretic postulates. Note that this derivation is generic, because the arguments we used in this section rely on neither the specific form of the metric nor any symmetries the fields may have.

What else can the information theoretic formalism tell us about the holographic principle? First, the holographic principle cannot be applied to a general surface that is not a causal horizon. The range of application of the principle was a long standing problem. Second, according to postulate 2, there should be a finite length scale l , and a finite area l^2 that can contain a bit of information. Thus, the total entropy S that the surface $\partial\Omega$, and hence the bulk Ω , can have is proportional to the surface area A ;

$$S = \frac{A}{l^2}. \quad (9)$$

The area law naturally emerges too. Third, it implies that the black hole entropy represents the uncertainty of field configurations inside the black hole horizon.

How can we relate bulk physics with boundary physics? To demonstrate the plausibility of theorem 1, I derive Witten's prescription of the holographic principle as an example. (Assume that the scalar field ϕ has a Lagrangian

$L_0 = \sqrt{-g}(\nabla^\mu \phi \nabla_\mu \phi - V(\phi)).$ The conjecture says

$$Z_{\partial\Omega}[\phi_0] = Z_\Omega[\phi|_{\partial\Omega} = \phi_0], \quad (10)$$

where $Z_{\partial\Omega}[\phi_0] \equiv \langle \exp(-\int d^d X \phi_0 \lambda) \rangle_{\partial\Omega}$ is the generating functional on $\partial\Omega$ with ϕ_0 as a source for a boundary field λ . $Z_\Omega[\phi|_{\partial\Omega} = \phi_0]$ is the partition function for the bulk field ϕ on Ω which approaches ϕ_0 at the boundary.

In Sec. II, we used only one nontrivial constrain on E . More generally, other constraints may exist regarding boundary field values $\phi = \phi_0$ on $\partial\Omega$, which the outside observer Θ_O can measure in principle. (When it is impossible to assign a field value on $\partial\Omega$, one may consider a stretched horizon instead of the horizon itself.) Thus, the field value ϕ_0 and its derivatives at each point on the boundary $\partial\Omega$ could be the maximal information (besides E and the form of $H(\phi)$) that the observer Θ_O can measure or change to influence the physics in Ω and that constrains the probabilities for the field in Ω . For simplicity, we assume a Dirichlet boundary condition.

Alternatively, one can describe the bulk physics by using only the quantities defined on the boundary. Imagine that there is only one boundary field λ that has an action S_λ and an interaction term $\phi_0 \lambda$. Then, the partition function for the boundary field is

$$Z_\lambda[\phi_0] = \frac{\int D\lambda e^{-S_\lambda} e^{-\int_{\partial\Omega} \phi_0 \lambda}}{\int D\lambda e^{-S_\lambda}}, \quad (11)$$

which is just $Z_{\partial\Omega}[\phi_0]$. We have set $Z_\lambda[\phi_0 = 0] = 1$.

The effective number of DOF of λ should be equal to that of ϕ , because theorem 1 implies that the boundary physics with $(S_\lambda(\lambda), E, \phi_0)$ has all the information about the bulk field having $(E, \phi_0, H(\phi))$ as parameters. For a given E and H , the classical field ϕ_0 is the only free parameter describing both of the bulk partition function and the boundary partition function. The partition function for a thermal system should contain all the information of the system. Since λ should describe the physics of ϕ , $Z_\lambda[\phi_0] = Z_\Omega[\phi|_{\partial\Omega} = \phi_0]$ should hold for λ having ϕ_0 as a source. Here, S_λ is not arbitrary but should be such that $Z_\lambda[\phi_0]$ well reproduces $Z_\Omega[\phi|_{\partial\Omega} = \phi_0]$. In other words, there is a duality mapping $(\phi_0, H(\phi)) \longleftrightarrow (\phi_0, S_\lambda(\lambda))$. λ could be hypothetical (mathematical) rather physical. In short, Witten's prescription is a natural consequence of the information theoretic formalism. However, the derivation above does not guarantee the existence of (λ, S_λ) for arbitrary bulk fields. Since a description is not a physical rule, the derivation is enough for our purpose.

Of course, the saddle point approximation of the bulk partition function becomes

$$Z_\lambda[\phi_0] \simeq \exp(-I_E(\phi)), \quad (12)$$

where $I_E(\phi)$ is the Euclidean classical action with the boundary condition $\phi|_{\partial\Omega} = \phi_0$ in the curved spacetime, as usual. Then, Witten's prescription yields a relationship between QFT and gravity.

To be concrete, let us consider a derivation of the prescription for the Rindler metric in detail as an example. To show the equivalence we divide the surface $\partial\Omega$ into N_j small patches and discretize the bulk field ϕ . We also assume that the field satisfies a Dirichlet boundary condition at the horizon. By repeating the calculation leading to Eq. (4) with additional constraints on the expectation values σ_j of the boundary field ϕ_0 at the j -th patch ϕ_{0j} ,

$$\langle \phi_{0j} \rangle \equiv \sum_i P[\phi_i] \phi_{0j}(\phi_i) = \sigma_j, \quad (j = 1 \cdots N_j), \quad (13)$$

one can easily obtain the probability distribution

$$P[\phi_i] = \frac{1}{Z} \exp \left[-\beta H(\phi_i) - \sum_{j=1}^{N_j} \lambda_j \phi_{0j}(\phi_i) \right], \quad (14)$$

where $\phi_{0j}(\phi_i)$ represents a boundary field value at the j -th patch corresponding to a specific bulk field configuration ϕ_i , and λ_j is the Lagrange multiplier field at patch j . Of course $Z = \sum_{i=1}^n \exp \left[-\beta H(\phi_i) - \sum_{j=1}^{N_j} \lambda_j \phi_{0j}(\phi_i) \right]$ now. The index j denotes the position on $\partial\Omega$ and shall be promoted to the d -dimensional coordinate X in the continuum limit. Since the number of independent λ_j values is N_j and λ_j couples to the boundary field, we can naturally think of λ_j as another scalar field on the boundary $\partial\Omega$. Taking a continuum limit and repeating the procedure leading to Eq. (8) we obtain

$$Z_Q^E[\sigma] = N_1 \int D\phi(x) \exp \{ - \int d\tilde{r} d\tilde{t} dx_\perp [L_0(\phi) + \lambda(X)\sigma(X)] \}, \quad (15)$$

where L_0 is the Euclidean Lagrangian and $\lambda(X)$ and $\sigma(X)$ are promoted continuous versions of λ_j and σ_j , respectively.

Note that this term is just the right-hand side of Eq. (10), $Z_\Omega[\phi|_{\partial\Omega} = \sigma]$ with $\langle\phi_0(X)\rangle = \sigma(X)$ identified as a classical boundary value for ϕ at $\partial\Omega$. This identification is physically reasonable because, strictly speaking, our theory and ordinary QFT contain no genuine classical field. The classical field is an approximate concept valid only in a specific limit.

Next, we need to show that $Z_Q^E[\sigma]$ is equivalent to $Z_{\partial\Omega}[\sigma]$. Since $\sigma(X)$ is a c-function defined on the boundary, it has no-dynamics on $\partial\Omega$. Thus, one can think of $\sigma(X)$ as a source function linked with some boundary field. Considering the action in Z_Q^E , the simplest candidate for the boundary field is the Lagrange multiplier field $\lambda(X)$ itself. This dual field may have an action $S_\lambda(\lambda)$ on $\partial\Omega$. Therefore, the partition function for the boundary field is

$$Z_\lambda[\sigma] = \frac{\int D\lambda e^{-S_\lambda} e^{-\int_{\partial\Omega} \sigma \lambda}}{\int D\lambda e^{-S_\lambda}}, \quad (16)$$

which is $Z_{\partial\Omega}[\phi_0]$.

Within the conventional QFT formalism, it seems impossible to prove the general equivalence of $Z_\lambda[\sigma]$ and $Z_Q^E[\sigma]$ because of the difference in their spacetime dimensions. Here, we must recall theorem 1. According to the theorem, all the information in the bulk should be contained in the boundary, and σ should be the only messenger available (except for E) between the bulk and the boundary. (We also need to link S_λ to H .) In other words, regarding ϕ in Ω and $\partial\Omega$, the outside observer can change only σ . Furthermore, the relevant fields describing the effectively same system (the bulk and boundary scalars) are ϕ and λ . Thus, the two partition functions as functionals of σ should be equal, i.e., $Z_\lambda[\sigma] = Z_Q^E[\sigma]$ and Witten's prescription Eq. (10) holds for the Rindler metric. The generating functional and the Euclidean nature of Witten's prescription arise naturally in the formalism.

IV. CONCLUSIONS

In summary, this paper shows that the holographic principle, like quantum mechanics and gravity, is not fundamental but emerges from information loss at causal horizons. The derivation is generic because we assumed neither supersymmetry nor string theory. This suggests the universality of the holographic principle applied at causal horizons and validates the application of the principle to other quantum systems such as condensed matter. The principle is intimately related to quantum mechanics. The derivation of the holographic principle in this paper is not a simple transformation of the principle to the postulates, because with the postulates one can derive quantum mechanics and Einstein's gravity as well as the principle. Since quantum mechanics and holography originate in information loss at causal horizons, information seems to be the common root of physics. This could open a new route to unifying gravity and quantum mechanics.

In our future work, we need to verify the equivalence of the information theoretic formalism and QFT in a more generic curved spacetime. To check the usefulness of this formalism, it is desirable to show the relationships between partition functions for other spacetimes, especially the AdS/CFT correspondence.

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